Catharsis Time Mathematical Models

Francisco Tamayo-Enríquez
ITESM Lean Six Sigma Program

Dr. Alberto Hernández Luna
ITESM Lean Six Sigma Program Director

Eric D. Smith
Systems Engineering Program
Research Institute for Manufacturing and Engineering Systems
Industrial, Manufacturing and Systems Engineering Department
University of Texas at El Paso
esmith2@utep.edu

Ramakanth Gona
Research Assistant
Industrial, Manufacturing and Systems Engineering Department
University of Texas at El Paso

Abstract:
A major problem in organizations is facing the existence of externalities in their complex engineering systems (CES) (Govindaraj, 2004). Once externalities exist, they may give rise to dangerous latent errors and active errors. Latent errors are those which are difficult to observe and difficult to remove, and active errors are those that are easily detected and removed (Kohn et al., 2000, p.210). Thus there should be a series of internalization steps to be taken by organizations in order to eliminate these type of errors and become aware of costs created by these errors in complex engineering systems. In this paper, we discuss the major role that "Catharsis Time" plays in complex engineering systems. Concepts and equations that model the relationship between catharsis time and costs of effects are developed in this work.

Introduction:
Fuentes (1997, p.119) elegantly defines Catharsis Time as "the time required to transform catastrophe into knowledge." It is proposed in this work that the competitiveness of complex engineered systems depends on their "Catharsis Time." Catharsis Time is here defined as the time in which an organization can achieve the internalization and permanent solving of errors. A more complete definition of Catharsis Time can be stated as the time it takes the complex system to convert failure into applied and reusable knowledge. Catharsis time depends on the magnitude of the problem. Catharsis Time is a simple metric that can be used to evaluate the effectiveness of a complex system in internalizing its errors.

Towards optimization of Catharsis Time

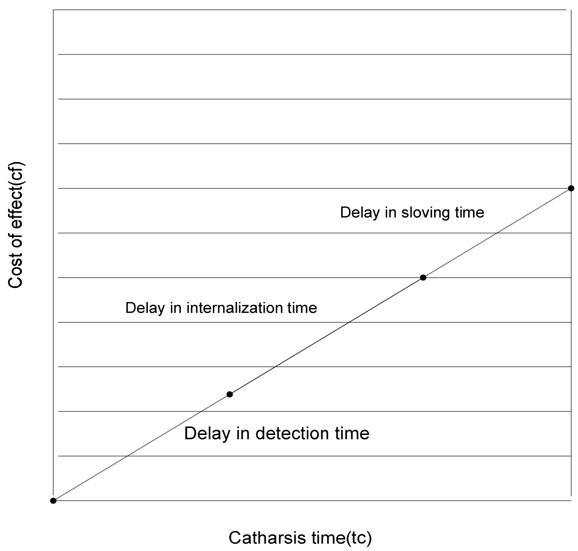
In its simplest form a linear relationship can be established between the Cost of Effect (c_f) and the Catharsis Time (t_c) of the complex engineered system. Equation 1 shows a function of the cost of negative effect in function of catharsis time.

$$c_f(t_c) = b_f t_c \qquad (1)$$

Where:

c_f = Cost of catharsis as a function of catharsis time
t_c = Catharsis time, from the occurrence of a negative action to its solution
b_f = Cost of the effect per time unit ($ / time)

Catharsis Time (tc) includes the
Delay in Detection Time (δ_1), Delay in Internalization Time ($_2$) and
Delay in Solving Time (δ_3).
See Graph 1.



Graph 1: Graphical representation of the Cost of Effect per time unit of Catharsis Time (tc)

On the other hand, it is reasonable to expect that reducing Catharsis Time (t_c) has a cost. It is proposed that in general this cost is significant, since a series of devices, alarms, decisions and procedures need to be implemented.

An exponential function can be used to model the Cost of Reaction or cost of reducing each time unit of Catharsis Time (cr) at the Catharsis Time (tc) of the complex engineered system. Equation 2: Function of the Cost of reducing Catharsis Time:

$$c_r(t_c) = b_r e^{-\alpha t_c} \qquad (2)$$

Where:

$c_r(t_c)$ = Cost of reaction at a given value of t_c (in $)

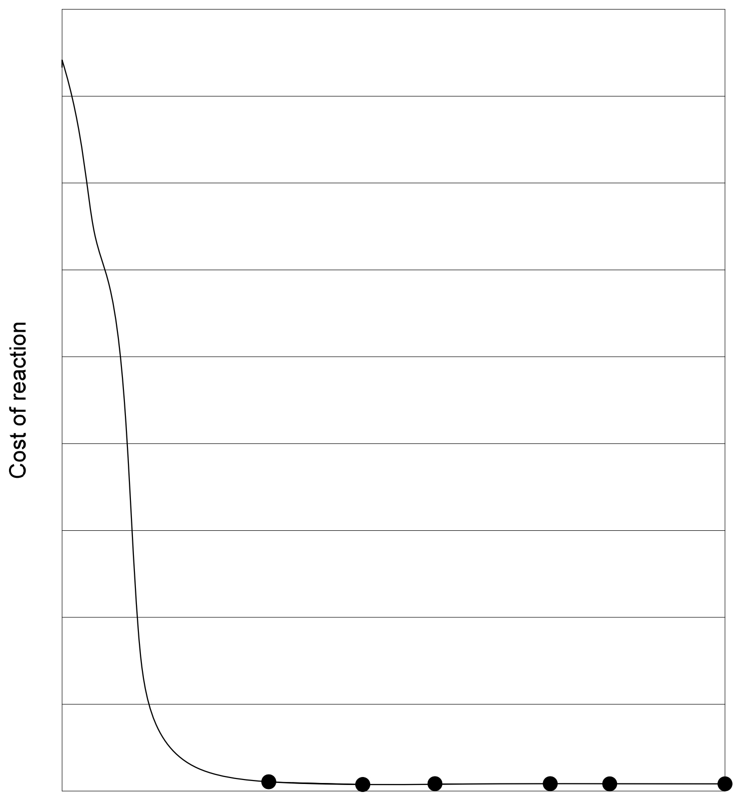
= Cost of reducing catharsis time to a given value of t_c (in $)

t_c = Catharsis time, from the occurrence of a negative action to its solution

b_r = Constant value that defines the cost of reducing Catharsis Time to 0 (in $, >0)

α= Constant that shapes the cost of reaction curve (no units, >0)

By using this formula we assume that the complex system would be able to react by itself without additional costs to relatively large Catharsis Time values. See graph 2.



Graph 2: As Catharsis Time increases the system is able to solve without additional resources

Substituting the previous values in the Total Cost of Error Function, we can establish the following function. Equation 3: Definition of Total Cost of Error in function of Catharsis Time.

$$c_e(t_c) = b_f t_c + b_r e^{-\alpha t_c} + c_{eo} \quad (3)$$

Where:

c_e = Total Cost of Error in function of Catharsis Time (in $)

t_c = Catharsis time, from the occurrence of a negative action to its solution

b_f = Constant value of the cost per time unit of the effect (in $, >0)

b_r = Constant value that defines the cost of reducing Catharsis Time to 0 (in $, >0)

α= Constant value that shapes the Cost of Reaction curve (no units), >0
c_{eo}= Fixed Cost of Error (in $)

We can establish a Catharsis Time that optimizes this previous Total Cost of Error Function by obtaining its derivative:

$$c_e(t_c) = b_f t_c + b_r e^{-\alpha t_c} + c_{eo}$$

$$\frac{\delta c_e(t_c)}{\delta t_c} = b_f - \alpha b_r e^{-\alpha t_c}$$

Setting to zero the previous function in order to find an Optimal Value of Catharsis Time (t_c *)

$$b_f - \alpha b_r e^{-\alpha t_c^*} = 0$$

$$b_f = \alpha b_r e^{-\alpha t_c^*}$$

$$\frac{b_f}{\alpha b_r} = e^{-\alpha t_c^*}$$

$$\ln\left(\frac{b_f}{\alpha b_r}\right) = -\alpha t_c^*$$

$$t_c^* = -\alpha^{-1} \ln\left(\frac{b_f}{\alpha b_r}\right)$$

$$t_c^* = \begin{cases} -\alpha^{-1} \ln\left(\dfrac{b_f}{\alpha b_r}\right) \text{if } bf < \alpha br \\ 0 \text{ if } bf \geq \alpha br \end{cases} \qquad (4)$$

Equation 4: Optimal Catharsis Time

Where:
t_c*= Optimal Catharsis Time to minimize the Cost of Error C_e (in time units)
b_f = Constant value of the cost per time unit of the effect (in $, >0)
b_r = Constant value that defines the cost of reducing Catharsis Time to 0 (in $, >0)
α= Constant value that shapes the Cost of Reaction curve (no units), >0

A numerical example is presented. An Active Agent manages a complex system with the following values:

Value of Cost Effect slope (b_f):	\$2.97 per day
Cost of setting Catharsis Time to zero (b_r):	\$31.67
Value for the Cost of Reaction curve (α):	1
Constant cost of error (c_{eo}):	\$10
Current Catharsis Time (t_c)	4 days

Using Equation 3, the Total Cost of Error of this case can be determined:

$$c_e(t_c) = b_f t_c + b_r e^{-\alpha t_c} + c_{eo}$$

$$c_e(4) = (2.97)(4) + (31.67)e^{-(1)(4)} + 10 = \$22.46$$

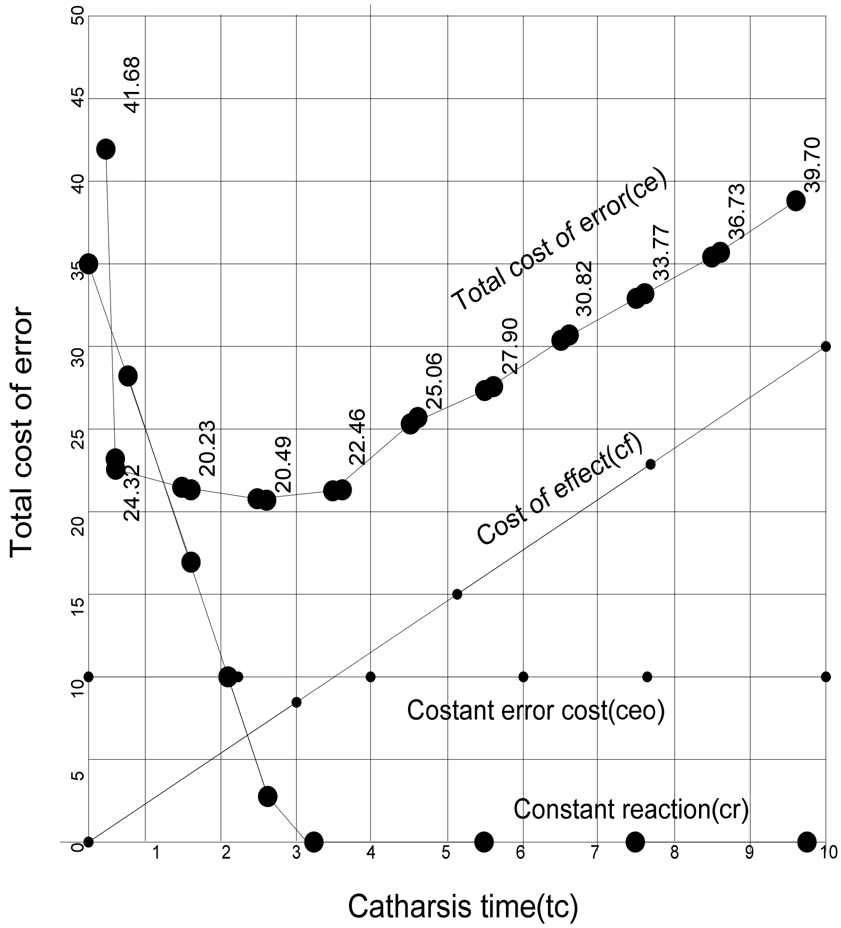
The Active Agent is interested in minimizing this cost in order to maximize his profits. Since b_f value is smaller than αb_r value, we can expect an optimal Catharsis Time (t_c) larger than zero. When the costs of the effects per time unit (c_f) defined by b_f are larger than the costs of setting the Catharsis Time to zero, it makes economic sense to invest in eliminating Catharsis Time at all (which may be the case with a preventive measure or an error-proof device that eliminates the possibility of a catastrophic error occurrence and therefore sets the Catharsis Time value to zero).

According to Equation 4, the optimal value of Catharsis Time (t_c*) for this system is:

$$t_c^* = -\alpha^{-1} \ln\left(\frac{b_f}{\alpha b_r}\right)$$

$$t_c^* = -1^{-1} \ln\left(\frac{2.97}{(1)(31.67)}\right) = -\ln(.0938) = 2.36$$

Under these conditions, the optimal Catharsis Time of this complex system is to be set up in order to be able to solve errors in 2.36 days. A chart of Total Cost of Error (c_e), Costs of Effect (c_f) and Cost of Reaction (c_r) in function of Catharsis Time values is presented. See Graph 3.



Graph 3: Costs of Error in function of Catharsis Time (tc)

Equation 3 and Equation 4 are only valid for the case where a single kind of error is expected (this equation could be useful for establishing the optimal Catharsis Time of a single sensor, for example). As discussed previously, this is not the case in complex engineered systems.
A cost model for two different errors is established in Equation 5.

$$c_{e2}(t_c) = (b_{f1} + b_{f2})t_c + (b_{r1} + b_{r2})e^{-\alpha t_c} + c_{eo1} + c_{eo2} \quad (5)$$

Equation.5: Definition of Total Cost of Two Errors in function of Catharsis Time

Where:

$c_{ei}(t_c)$ = Total cost of error in function of Catharsis Time (in \$) for i errors, i = [1,2]
t_c = Catharsis time, from the occurrence of a negative action to its solution
$b_{f\,i}$ = Constant value of the cost per time unit of the effect of error i (in \$), >0
$b_{r\,i}$ =Constant value that defines the cost of setting Catharsis Time to zero for error i (in \$), >0
α= Constant value that shapes the cost of setting Catharsis Time curve (no units), >0
$c_{eo\,i}$ = Fixed Cost of Error i (in \$)

Following the same procedure used previously for determining the optimal Catharsis Time in Equation 4, the optimal Catharsis Time for a model considering two errors would be

$$t_c^{*} = \begin{cases} -\alpha^{-1} \ln\left(\dfrac{b_{f1}+b_{f2}}{\alpha(b_{r1}+b_{r2})}\right) \text{if}\,(b_{f1}+b_{f2}) < \alpha(b_{r1}+b_{r2}) \\ 0\,\text{if}\,(b_{f1}+b_{f2}) \geq \alpha(b_{r1}+b_{r2}) \end{cases} \tag{6}$$

Equation 6: Optimal Catharsis Time for 2 errors

And its general form in order to consider n-errors would be

$$t_{cn}^{*} = \begin{cases} -\alpha^{-1} \ln\left(\dfrac{\sum_{i=1}^{n} b_{fi}}{\alpha \sum_{i=1}^{n} b_{ri}}\right) \text{if} \sum_{i=1}^{n} b_{fi} < \alpha \sum_{i=1}^{n} b_{ri} \\ 0 \text{ if} \sum_{i=1}^{n} b_{fi} \geq \alpha \sum_{i=1}^{n} b_{ri} \end{cases} \tag{7}$$

Equation 7: Optimal Catharsis Time considering n different errors

Where:

t_{cn}* = Optimal Catharsis Time to minimize the Cost of Error C_e for n different errors (in time units)

b_{fi} = Constant value of the cost per time unit of the effect of error i (in \$), >0

b_{ri} = Constant value that defines the cost of setting Catharsis Time to zero for error i (in \$), >0

α= Constant value that shapes the cost of setting Catharsis Time curve (no units), >0

Equation 7 could be useful for systems where the total number of errors is known and costs could be determined more or less accurately. However, complex engineered systems potentially have an infinite number of system states, and therefore, it is expected that an infinite (or at least unknown) number of different error could occur.

A more general Total Cost of Error is proposed for complex engineered systems with an unknown number of errors in Equation .8. In this model, the cost of having zero Catharsis Time is infinite, since the complex would require being infinitely resourceful in order to immediately react to any possible failure. However, this cost rapidly diminishes with larger values of Catharsis Time, since the complex system has now time to design reaction strategies and assign resources. Equation 8: Total Cost of Error in function of Catharsis Time for an unknown number of errors:

$$c_{es}(t_{cs}) = b_f t_{cs}^{\beta} + b_r t_{cs}^{-\alpha} + c_{eo} \qquad (8)$$

Where:

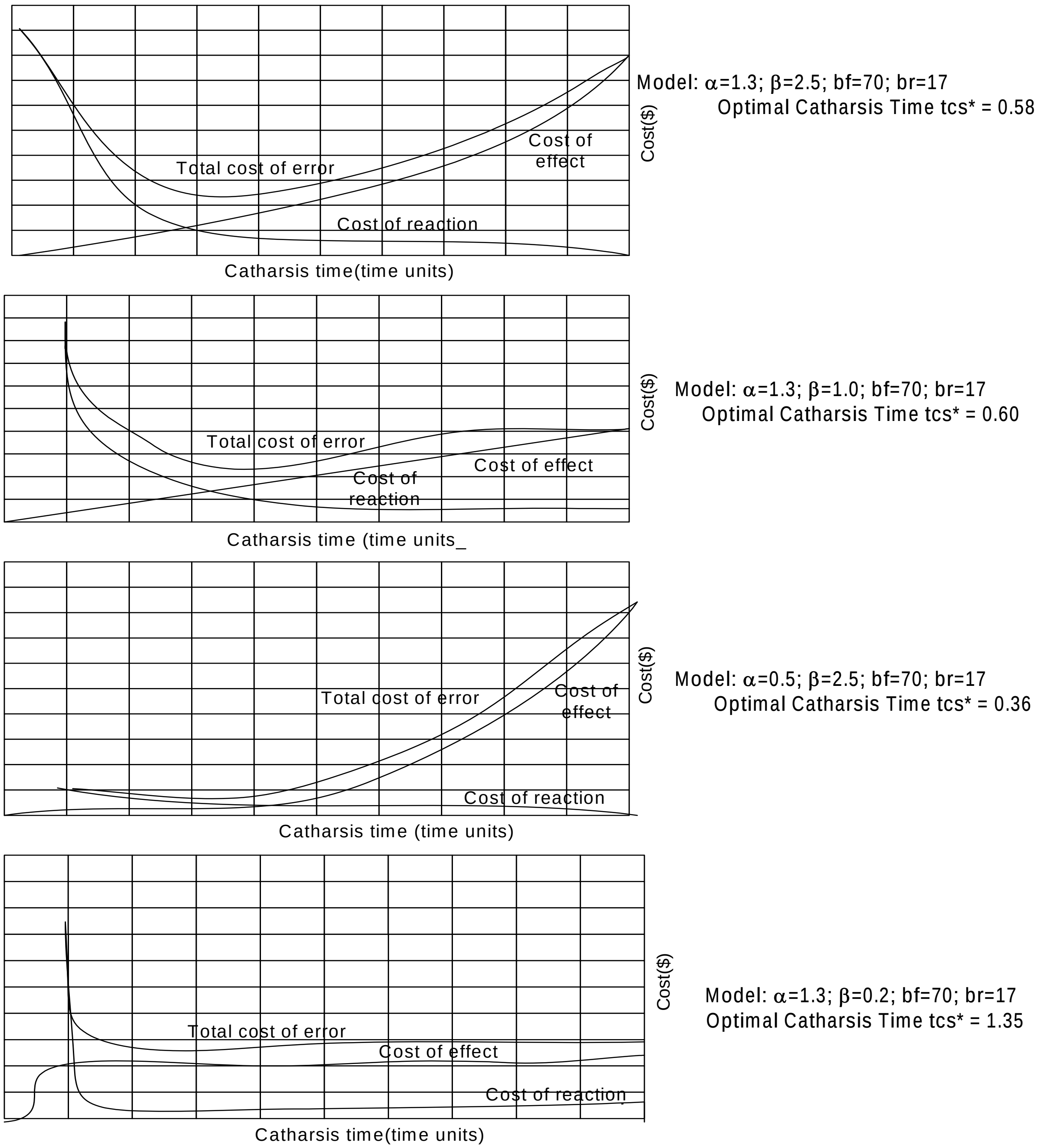
$c_{es}(t_{cs})$ = Total cost of unknown number of errors in function of Catharsis Time (in $)
t_{cs} = Catharsis Time. Time from the occurrence of a negative action to its solution (in time units)
b_f = Constant value of the cost per time unit curve of the effect (in $, >0)
b_r = Constant value of cost curve of setting Catharsis Time to a certain value (in $, >0)
α= Constant value that shapes the cost of setting Catharsis Time curve (no units, >0)
β= Constant value that shapes the cost per time unit curve (no units, >0)
c_{eo}= Fixed Cost of Error (in $, >0)

Following the same procedure used previously for determining the optimal Catharsis Time in Equation 4, the optimal Catharsis Time for a general cost model considering an unknown number of errors would be:

$$t_{cs}^{*} = \left[\frac{\alpha b_r}{\beta b_f} \right]^{(\alpha+\beta)^{-1}} \qquad (9)$$

Equation 9: Optimal Catharsis Time for a General Cost Model considering an infinite number of errors

Different models can be constructed by changing the parameters in Equation 7. Some graphical examples are presented in Graph 4.



Graph 4: Different models can be constructed by changing the parameters in Equation 7.

Using Catharsis Time to assess different experimental strategies

The response (z) of a Complex Engineered System is defined by the following equation:

$$z(x,y)=\frac{1}{20}\left(-.046\mathrm{x}^3+.43\mathrm{x}^2+.37\mathrm{x}+\frac{1.08+.093\mathrm{y}}{1-.2567y+.02y^2}+2.67\right) \tag{10}$$

Equation 10: Response variable to optimize

Where

X	=	control variable [0, 1, 2, 3, .10]
Y	=	control variable [0, 1, 2, 3, 10]
Z	=	response variable to maximize. A-z value <0.95 is consider an error

Values of (x, y) are assumed to change in integer values between [0, 10]. This formula is unknown to the decision-maker. Moreover, there is some randomness in the model, due to variability, measurement error, etc. For reference, Table 1 with all the valid results is presented.

	0	1	2	3	4	5	6	7	8	9	10
0	0.1875	0.2252	0.2921	0.3744	0.4583	0.5300	0.5757	0.5816	0.5339	0.4188	0.2225
1	0.2103	0.2480	0.3149	0.3972	0.4811	0.5528	0.5985	0.6044	0.5567	0.4416	0.2453
2	0.2452	0.2829	0.3498	0.4321	0.5160	0.5877	0.6334	0.6393	0.5916	0.4765	0.2802
3	0.2993	0.3370	0.4039	0.4862	0.5701	0.6418	0.6875	0.6934	0.6457	0.5306	0.3343
4	0.3811	0.4188	0.4857	0.5680	0.6519	0.7236	0.7693	0.7752	0.7275	0.6124	0.4161
5	0.4903	0.5280	0.5949	0.6772	0.7611	0.8328	0.8785	0.8844	0.8367	0.7216	0.5253
6	0.5890	0.6267	0.6936	0.7759	0.8598	0.9315	*0.9772*	*0.9831*	0.9354	0.8203	0.6240
7	0.6062	0.6439	0.7108	0.7931	0.8770	0.9487	*0.9944*	*1.0003*	*0.9526*	0.8375	0.6412
8	0.5363	0.5740	0.6409	0.7232	0.8071	0.8788	0.9245	0.9304	0.8827	0.7676	0.5713
9	0.4430	0.4807	0.5476	0.6299	0.7138	0.7855	0.8312	0.8371	0.7894	0.6743	0.4780
10	0.3656	0.4033	0.4702	0.5525	0.6364	0.7081	0.7538	0.7597	0.7120	0.5969	0.4006

Table 1: Matrix of the 121 possible values of the function z(x, y) and contour plot

Several strategies may be followed by a decision-maker who is willing to minimize the total costs of achieving an optimal response by using some sort of evolutionary process. It is proposed in this work that the procedure and equations presented previously could be useful in order to support the comparison and selection of different processes.

Hyper geometric Distribution

Although generally inefficient, a possible process to use is to make a random selection of (x, y) values and evaluate the function in this level. If the desired result is achieved, the process finalizes. If it is not, a different selection of (x, y) values is done. 'Different' implies that values already evaluated and determined to be non-optimal would be discarded.

In this particular case, two conditions are set:

1. The decision-maker would accept as optimal values that are greater than or equal to 0.95
2. Exactly 121 possible combinations of (x, y) values are feasible

The previous conditions allow the use of the hyper geometric distribution. The hyper geometric distribution is a discrete probability distribution that describes the number of successes in a sequence of n draws from a finite population without replacement and is presented.

$$f(k;N,m,n) = \frac{\begin{pmatrix} m \\ k \end{pmatrix}\begin{pmatrix} N-m \\ n-k \end{pmatrix}}{\begin{pmatrix} N \\ n \end{pmatrix}} \tag{11}$$

Equation 11: Hyper geometric function

F (k; N, m, =Hyper geometric distribution. Probability of exactly k defined objects (m) in a sample of n

N =Total of population that may be selected
m =Number of objects defined by a certain characteristic
n =Sample size to be taken, without replacement of identified objects
k =Exact number of defined objects (m) to be obtained

The following procedure was followed in order to obtain an initial comparison:

1. Define N. Exactly 121 possible combinations of (x, y) values are feasible. N = 121
2. Define m. As stated previously the decision-maker would accept as optimal z values that are greater than or equal to 0.95 and from Figure 13 we know that 5 combinations of (x, y) values give z values that satisfy this condition. m = 5.
3. Define k. We want to find out the number of experiments required in order to obtain z value $\geq$ 0.95. Therefore, k = 0 in order to find [1-p(k)]
4. Define n. Different values of n would be used in order to calculate the number of experiments required in order to obtain an m.
5. Calculate hyper geometric function. For example, for n = 10, the following iteration sequence was established in Table 2.

Iteration	1
K	0
N	10
M	5
N	121
Hyper geometric	64.47%
1- Hyper geometric	35.53%

Table 2: representation of iteration sequence

In a sample of 10 (x, y) values without replacement, there is a 64.47% of probability of not obtaining an optimal value of z. On the other hand, there is a 35.53% of obtaining one. A probability tree of this case is presented in Figure 1.

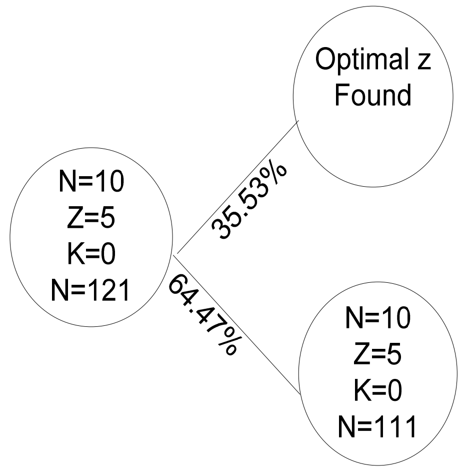


Figure 1: Probability tree of the hyper geometric distribution
of a sample of 10 (x, y) values, first iteration

If the selected pairs do not contain an optimal z value (p = 35.53%), a second iteration could be performed using the remaining 111 untested pairs. The six following iterations are presented graphically with the probability tree shown in Figure 2.

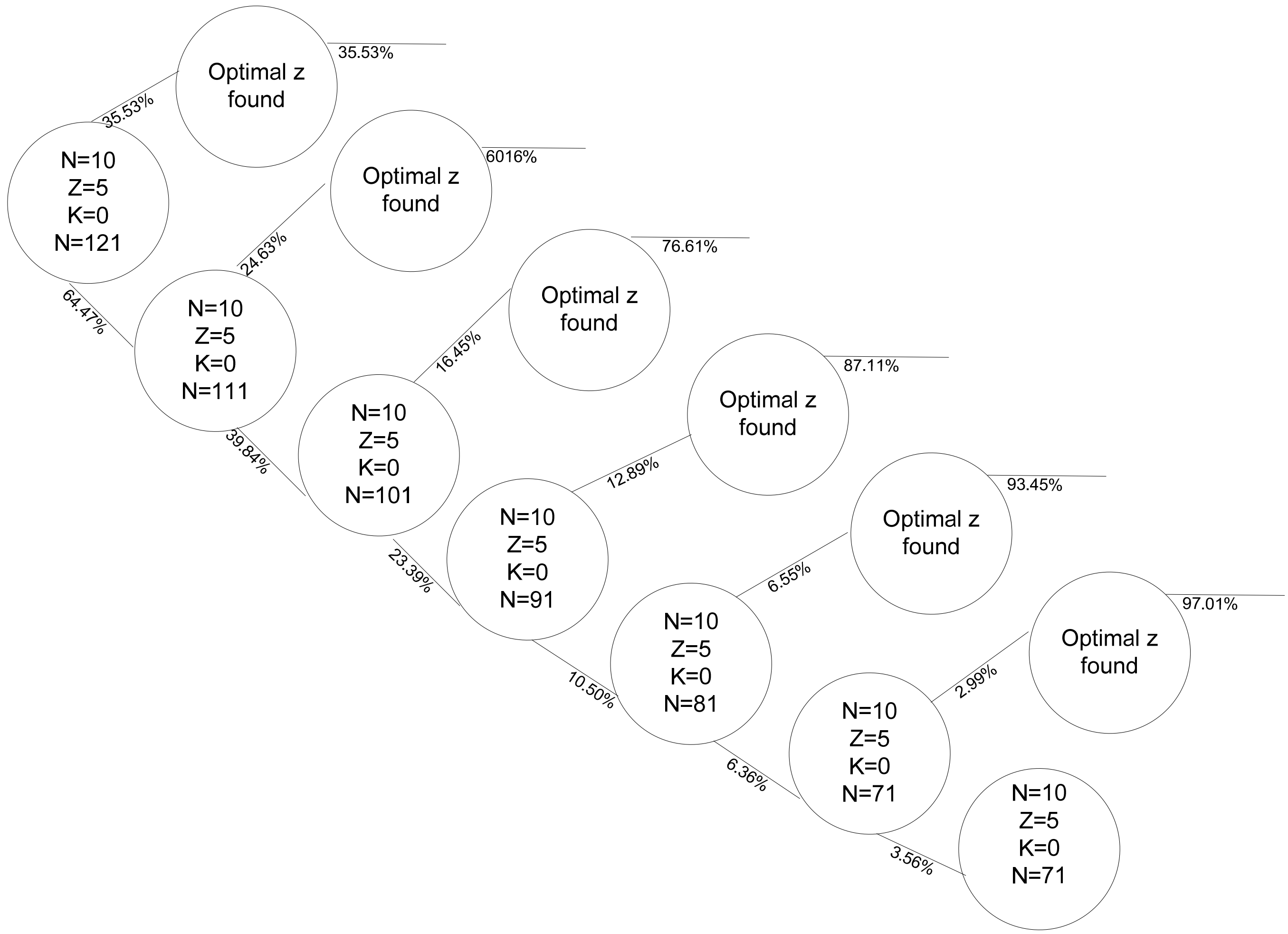


Figure 2: Probability tree of the hyper geometric distribution
of a sample of 10 (x, y) values, sixth iteration

Six iterations are required in order to achieve a probability larger than 95% of obtaining an optimal z value. We can set an iteration time equivalent to 1 unit of Catharsis Time (t_c). Therefore, the Catharsis Time value for obtaining optimal z-values more than 95% of the cases with 10 samples of (x, y) is equal to:

$$tc(n=10) \approx (.3553)(1) + (.2463)(2) + (.1645)(3) + (.1050)(4) + (.0634)(5) + (.0356)(6)$$

$$tc(n=10) \approx 2.29$$

This Catharsis Time converges to 2.52 after 11 iterations, but since the process will be stopped after 6 iterations (the optimal values are not truly known by the decision-maker, so a stopping rule is required) the estimated 2.29 value would be used. A table with different values of n was prepared and its Catharsis Time (average number of Iterations) calculated in Table 3.

n	tc	Max itreations
60	0.97	1
50	1.07	2
40	1.12	2
30	1.17	2
25	1.35	3
20	1.44	3
15	1.72	4
12	2.00	5
11	2.05	5
10	2.29	6
9	2.36	6
8	2.64	7
7	2.94	8
6	3.27	9
5	3.87	11
4	4.76	14
3	6.05	18
2	8.82	27
1	17.16	54

Table 3: Catharsis Time and maximum number of iterations for different sampling n-values

It is known that the cost of running an experiment is of $40. Therefore, the Cost of Reaction (c_r) would be:

$$c_r(t_c) = 40(test)$$
$$c_r(t_c) = 40(t_c)(n)$$

The cost of the effect is of $150 per unit of time. The Cost of Effect (c_f) would be a linear function in function of Catharsis Time.

The Total Cost would be

$$c_e(t_c) = 40(test) + 150t_c \qquad (12)$$

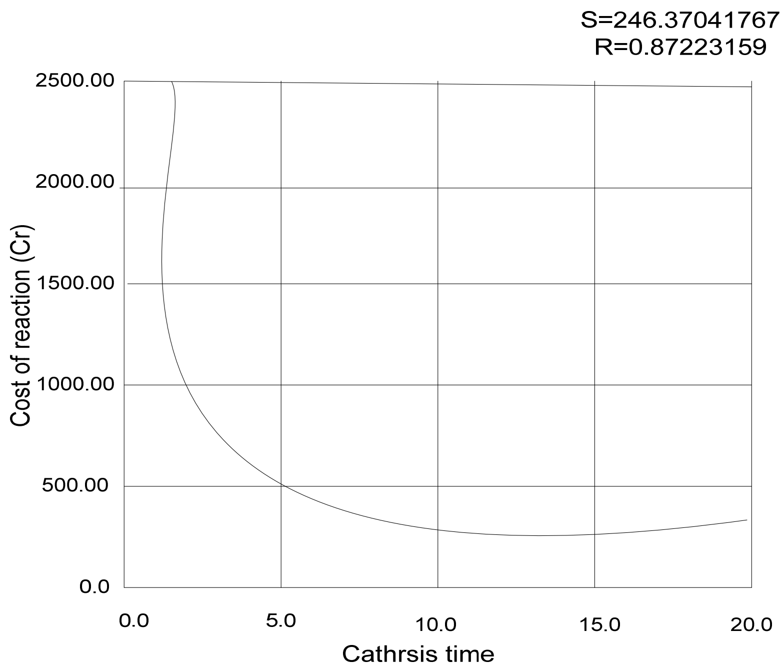
Equation 12: Total Cost in function of tests and Catharsis Time

n	Max iterations	Cost of reaction	Cost of effect	tc	Total cost
60	1	2328.00	145.50	0.97	2473.50
50	2	2140.00	160.50	1.07	2300.50
40	2	1792.00	168.00	1.12	1960.00
30	2	1404.00	175.50	1.17	1579.50
25	3	1350.00	202.50	1.35	1552.50
20	3	1152.00	216.00	1.44	1368.00
15	4	1032.00	258.00	1.72	1290.00
12	5	960.00	300.00	2.00	1260.00
11	5	902.00	307.50	2.05	1209.50
10	6	916.00	343.50	2.29	1259.50
9	6	849.50	354.00	2.35	1203.60
8	7	844.80	396.00	2.64	1240.80
7	8	823.20	441.00	2.94	1264.20
6	9	785.98	491.24	3.27	1277.22
5	11	774.00	580.50	3.87	1354.50
4	14	761.60	714.0	4.76	1475.60
3	18	726.00	907.50	6.05	1633.50
2	27	705.60	1323.00	8.82	2028.60
1	54	686.40	2574.00	17.16	3560.40

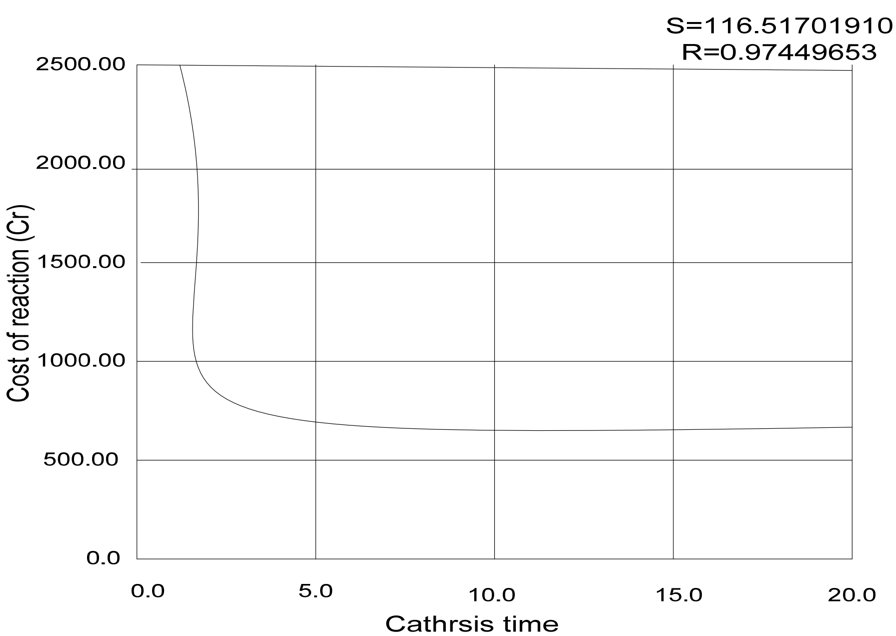
Table 4: Costs and Catharsis Time. Best value marked in line

The discrete nature of the options of sample sizes of n values of (x, y) makes it harder to determine an optimal value of Catharsis Time. Curve-fitting software was used in order to determine optimal values for the Cost of Reaction, since Cost of Effect is a linear function of Catharsis Time.

The standard model for the Cost of Reaction proposed in this work in Equation 8 is $Cr = b_r\, t_{cs}^{-\alpha}$. However in this specific case the correlation coefficient of this model is r=.8722 (br =1861.58, α=.72576). The plot of these values and the model built are presented in the Graph 5.



Graph 5: standard model for the Cost of Reaction



Graph 6: Heat-capacity model for Cost Reaction

The equation of this model is

$$C_r = 571.387 + 9.847t_c + 1519.217t_c^{-2}$$

The optimal value for the Total Cost was calculated with the same procedure that has been previously proposed within this work:

$$C_e(t_c) = 150t_c + 571.387 + 9.847t_c + 1519.217t_c^{-2}$$

$$\frac{\delta Ce(t_c)}{\delta t_c} = 150 + 9.84767 - (2)(1519.217)t_c\,*^{-3} = 0$$

$$t_c\,*^{-3} = \frac{159.84767}{3038.434}$$

$$-3\ln(t_c\,*) = \ln\left[\frac{159.84767}{3038.434}\right]$$

$$\ln t_c* = \frac{2.94}{3}$$

$$t_c* = e^{.9816}$$

$$t_c* = 2.66$$

Since the complete table was built, we know that the optimal value is 2.36 and that not all values of Catharsis Time are feasible. However, the 2.66 provides a good estimation of the zone of the optimal Catharsis Time for the model built using the hyper geometric distribution.

EVOP Algorithm using EVOPtimizer Software

Since Design of Experiments (DoE) "usually requires the process to be setup to operate with unusual combinations of process parameters, giving results which may be unpredictable and output uncertain to quality. Hence, conducting designed experiments on the actual process tends to cause disruption in the plant, and may be uneconomic. EVOP is one approach to this problem, which has been used where classical DoE is inappropriate (Sukthomya and Tannock, 2005, pp. 485-486)

"Using EVOP, carefully planned cycles of small changes are made to two or three process parameters as the process is operated, and the results observed. Disadvantages of cost, time and staff training, and the potential quality problems, have meant that EVOP, and approaches designed for it, have not been widely used (Sukthomya and Tannock, 2005, p. 488).

The EVOP model of the freeware program EVOPtimizer: Sequential Simplex Optimization Version 1.1.0 was used in order to calculate the average Catharsis Time (number of experiments required before reaching and optimal (x, y) value) and the average total costs. Also, two different strategies for setting up the initial simplex were compared:

a. Three random points (x,y)
b. A random center point and two adjacent corners (for example: center (5,4), corners (4,3) and (6,3))

Ten samples of both EVOP setup strategies were used.

a. Three random points

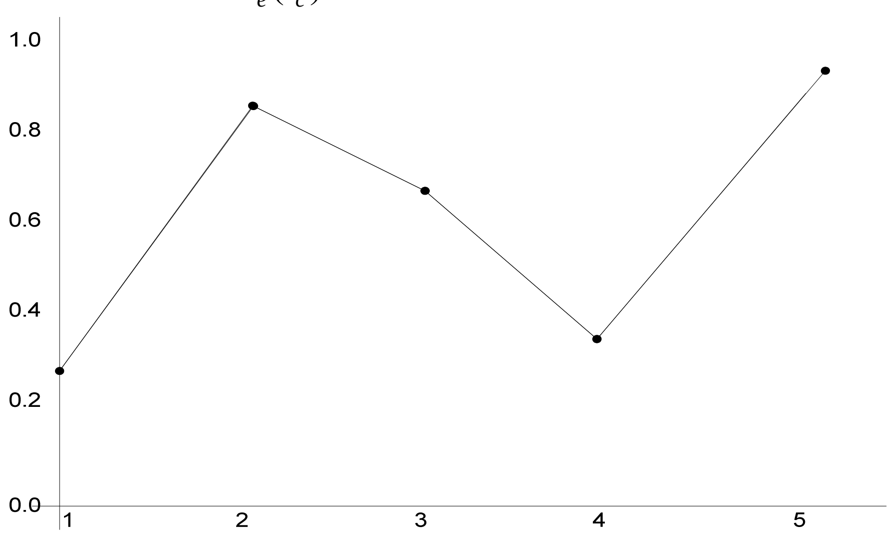
The total cost of each iteration was calculated according to Equation 12. An example of the table for trial 6 is presented TABLE 5. This table includes: Trial number 6; total cost $c_e(t_c)$ = 650; Catharsis Time 3; test or experiments 5; 3 x-y values for setup; and 2 x-y values of two iterations. A graphical representation for trial 6 is presented in Graph 7.

No	COST	Tc	EXP	X	Y	VAL	X	Y	VAL	X	Y	VAL	X	Y	VAL	X	Y	VAL
6	650	3	5	1	1	.248	9	7	.837	3	6	.776	10	10	.401	8	7	.963

Table 5: Example of table for trial 6

For trial 6, the total cost in function of number of tests and Catharsis Time is:

$$c_e(t_c) = 40(tests) + 150(t_c)$$
$$c_e(t_c) = 40(5) + 150(3)$$
$$c_e(t_c) = 200 + 450$$
$$c_e(t_c) = 650$$



Graph 7: Graph of EVOP of trial 6 (first 3 points are the setup t_c =1; Optimal response achieved in t_c = 3)

For this EVOP, 10 trials were run. The average number of tests was 11.70. The first 3 tests were required to setup the EVOP, so therefore are assumed to cost only 1 time unit for Catharsis Time. The average number of iterations (Catharsis Time) was 9.70. An outlier value is present, since a trial cycled itself. This was detected in iteration 17. A Catharsis Time = 20 was assigned to cycling runs.

The average total cost of this EVOP $c_e(t_c)$ = 1923. This is a costly model, compared to the best value of the hyper geometric model ($c_e(t_c)$ = 1203.60) or of the Student's t algorithm ($c_e(t_c)$ = 1106.70). The other problem of this model is its probability of not converging to a solution (one out of ten cases did not converge).

b. Center and two adjacent points

The random samples selected by the random function of Excel are presented in a partial view of a table see Table 6.

X	Y	VAL	X	Y	VAL	X	Y	VAL
2	9	0.548	1	10	0.403	3	8	0.723
7	2	0.639	6	3	0.687	6	1	0.599
1	1	0.248	0	0	0.188	2	0	0.292
2	0	0.292	1	1	0.248	3	0	0.374
6	1	0.599	5	2	0.588	5	0	0.53
1	1	0.248	2	2	0.35	0	0	0.188
4	1	0.481	3	2	0.432	3	0	0.374
2	5	0.595	3	4	0.568	1	4	0.419
0	6	0.589	1	7	0.644	0	5	0.49
2	9	0.548	1	10	0.403	3	10	0.553

Table 6: Ten random samples of center point and adjacent corners

The average number of tests was 10.80. The first 3 tests were required to setup the EVOP, so therefore are assumed to cost only 1 time unit for Catharsis Time. The average Catharsis Time was 8.80. Once again, two trials cycled themselves. A Catharsis Time = 20 was assigned for cycling trials.

With this EVOP, the average total cost is of $c_e(t_c)$ = 1752. It represents and improvement against the previous EVOP (cost = 1923) but it is still high compared to the best value of the hyper geometric model (cost = 1203.60) or of the Student's t algorithm (cost = 1106.70).

Since a demo version of the EVOP software is been used, the initial simplex cannot be changed. Therefore, always 3 tests will be used in the initial iteration (t_c =1). Therefore, the total cost could be represented with the following equation:

$$c_e(t_c) = 40(3) + 40(tc - 1) + 150tc$$
$$c_e(t_c) = 120 + 40tc - 40 + 150tc$$
$$c_e(t_c) = 80 + 190tc$$

In this case, the total cost of the EVOP is optimized when $t_c = 0$. It can be shown that in general, the total cost for EVOPs depends in the number of setup tests (n_1), the number of tests in each iteration (n_2) and the Catharsis Time.

$$c_e(t_c) = 40(n_1 - 1) + 150 n_2 tc$$

If we assume that EVOPs would move with only one test per iteration ($n_2 = 1$) a sensitivity of setup tests (n_1) can be performed in order to outperform the Student's t algorithm total cost $c_e(t_c) = 1106.74$. For example, the average Catharsis Time for a model with (n_1=4) should be smaller than:

$$c_e(t_c) = 40(n_1 - 1) + 150t_c$$
$$1106.74 < 40(3) + 150t_c$$
$$t_c < 6.57$$

Finding an optimal EVOP in order to minimize total costs (c_e) of this specific case is outside the scope of this work. Moreover, EVOP in this case was somehow unfairly assessed, since usually EVOP provides increasingly better feasible combinations, not good-bad points. However, it was useful to demonstrate how evolving algorithms could use Catharsis Time as a way of making decisions that relate the cost of reaction (C_r) with the cost of effect (C_f). It was also shown in this process that the costs equations developed previously could be changed, according to each situation. Although these equations are flexible and could be useful for optimizing a number of cases, in many others particular equations would be needed. On the other hand, the procedure of equations construction and Catharsis Time optimization presented in previous sections is proposed in this work as a useful strategy in order to relate and optimize costs of error with Catharsis Time.

Conclusion

The results presented are certainly case dependent. They depend in the relationship between the cost of reaction (c_r, cost of each experiment) and the cost of the effect (c_f, number of iterations). They also depend in the algorithm used and in the participation of the decision-maker who can provide 'good' starting points (Ghani, 1995) and stop a cycling iteration. This author proposes that sometimes it is better to use human ingenuity in order to reduce the dimensionality of the parameter space.

Since the total costs (cost of reaction + cost of effect) are case dependent, the closer the number of iterations of the EVOP/Genetic Algorithm is to the optimal Catharsis Time (t_c*) the better it would be. A challenge to the decision maker is to calculate the cost of reaction (c_r) and cost of effect (c_f) curves accurately.

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