

A Model for Nighttime Urban Illumination

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Abstract

The Army increasingly relies on night operations to accomplish its objectives. These night operations frequently require using Night Vision Goggles and other light-sensitive devices which are strongly affected by ambient lighting, a large component of which is urban. An urban illumination model is proposed for use in tactical decision aids and wargames which would allow for more accurate prediction of target acquisition ranges and increased realism in simulations. This model will build on previous research that predicts broadband brightness as a function of population and distance from the city center. Since city population and aerosols affect light distributions, the model is being extended and generalized for multiple city types and natural and man-made aerosols. An overview of the model along with future improvements will be presented.

Introduction

In order to predict target acquisition range of a given sensor observing a specific target under specified weather conditions, one must know the sensor and target characteristics, the weather conditions and the ambient illumination. The ambient illumination, which may be comprised of solar, lunar, galactic, and/or man-made lighting, strongly affects the ability of a sensor to “see” and, therefore, determine acquisition ranges. In night-time warfare, frequently the brightest sources of illumination are either from the moon or from urban areas. For Soldiers and pilots using night vision goggles (NVGs), operating near urban areas, care must be taken to not saturate NVGs by looking directly at brightly lit areas and effectively blinding the wearer. If the area of bright light is a small point (such as a single street light), then this will be seen as a ‘halo’ around the light, leading to a loss of contrast in the surrounding image. For mission planning, training purposes, and estimates of target acquisition ranges, it is necessary to have some type of model that will accurately take into account all of the above lighting sources and be able to deal with the associated problems such as ambient lighting conditions varying over the course of a mission. Furthermore, different lighting exists in different cities around the world whose population, and therefore the amount of cultural lighting, varies significantly.

Background

The slow but steady progress in NVG and Forward Looking Infrared (FLIR) technology has allowed the services to operate 24/7. Air and ground operations now occur frequently during nighttime hours, where detection by the enemy is reduced. NVGs can be used in both moon-lit and moonless conditions; the illuminance provided by the moon depends on its elevation and phase, and knowledge of these is important in planning NVG operations. However, as noted above, starlight can provide sufficient illumination to facilitate operations under moonless conditions. When weather conditions are such that neither the moon nor stars are visible, such as overcast clouds, NVGs can still operate using urban illumination. Aside from the dated model contained in the Tri-service Target Acquisition Weapons Software (TAWS) decision aid, there are no other models available to predict illumination levels from urban centers.

Therefore, there is a definite need to improve and update the TAWS model. The model currently under development, the Light, Urban Model Effects (LUME) model, will provide worldwide target acquisition city light levels for Soldiers using NVGs or other visual and near-visual sensors as a function of weather and distance from city center. It will be provided to target acquisition models (TAWS, NVESD's follow-on to Acquire), wargames, and simulations (IWARS, COMBATXXI) for improved target detection, Soldier effectiveness, and doctrine development.

Calculating the illumination from distant city lights requires 1) modeling the illumination source intensity and spectra, and 2) solving complex radiative transfer calculations including multiple scattering and aerosol and molecular absorption in a non-homogeneous atmosphere. Currently a number of models exist for predicting urban illumination as a function of distance from urban centers at visual wavelengths or portions thereof. These models range from the empirical^{1, 2, 3} and semi-empirical⁴⁻⁷, to research grade⁸, for the determination of brightness (direct + diffuse) of a city as seen by an observer through the atmosphere. These models were constructed primarily by members of the astronomical community to define the effects of light pollution on observatories and observation sites. However, in addition to an urban illumination we must also have information about the lighting sources – their intensity and angular and spectral characteristics.

The Sources: Illumination databases

We can consider the natural sky brightness to be a layer at infinity comprised of integrated starlight, diffused galactic light (a small contribution to the background glow by starlight reflected and scattered by interstellar dust near the galactic plane) and zodiacal light (a faint, roughly triangular, whitish glow seen in the night sky which appears to extend up from the vicinity of the sun along the ecliptic or zodiac) which covers the entire sky. The zodiacal light is produced by sunlight reflecting off dust particles which are present in the solar system and light due to airglow emission (a very weak emission of light caused by various processes in the upper atmosphere). Other sources include the moon and artificial light from urban centers and other manmade structures. On nights where the cloud cover obscures the natural sky brightness and contributions from the moon, the reflected light of urban centers frequently provide sufficient illumination for NVGs.

Accurate models of the natural sky brightness and lunar illumination exist; however, with the exception of TAWS, the author knows of no other models that attempt to integrate the effects of urban illumination.

The models previously mentioned use various illumination data bases and, for the most part, rely on population data to predict the urban illumination at remote locations. Cities frequently do not use the same type of lighting, leading to different city “spectral signatures”, and the type of lighting used is dependent upon differences in economic development and lighting practices⁵. There are a number of different approaches that can be used to mitigate this. First, the overall broadband brightness due to the city may be estimated by, as was originally done by Walker^{1,2}, measuring the brightness in the astronomical V band ($\sim 0.48 - 0.62 \mu\text{m}$) and then applying the population vs. intensity relation. This method can be modified and improved by identifying the spectral composition of urban light and subsequently estimating the percentage that each light source contributes in a typical city, subsequently allocating the total brightness with these percentages. This is the approach that has been used in TAWS⁹.

A second, somewhat similar approach has been used by Aubé⁸. He has acquired spectral data with the SAND¹⁰ spectrometer at various U.S. and Canadian locations, focused on key spectral lines representative of specific kinds of lighting (high pressure sodium, metal halide, and low pressure sodium). He then uses this data as input to his high-resolution research grade illumination model, using an iterative technique, comparing the model output to the observed SAND data, to determine the appropriate aerosol optical depth.

Recently, with the advent of digital satellite data, another approach may be viable. Using DSMP measurements coupled with Garstang’s illumination model⁷, Cinzano, et al.,¹¹ have produced a world atlas of artificial night sky brightness at sea level under clear skies - figure 1 presents a sample picture for the U.S. The map levels correspond to the artificial sky brightnesses as seen

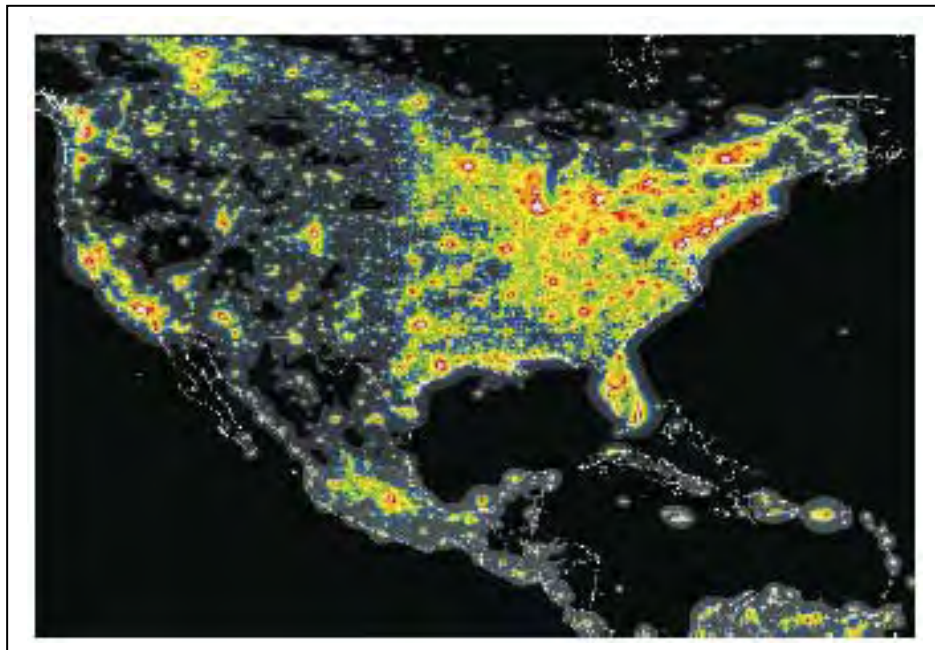


Figure 1. Artificial sky brightness as seen from sea level for the U.S.

from sea level in $V \text{ ph cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$, where the colors correspond to ranges of: 9.47×10^6 – 2.84×10^7 (blue), 2.84×10^7 – 8.61×10^7 (green), 8.61×10^7 – 2.58×10^8 (yellow), 2.58×10^8 – 7.75×10^8 (orange), 7.75×10^8 – 2.32×10^9 (red), $> 2.32 \times 10^9$ (white). Making appropriate assumptions regarding the light distribution, it may be possible to use this atlas as an input source.

The Models

The following section provides an overview of available models that are being considered as starting points for LUME.

By combining measurements of the artificial brightness of the night sky with the assumption that the brightness is proportional to a city's population Walker¹, was able to show that there was a relationship between distance from city center and city population. His result can be expressed as

$$P \propto D^{2.5}, \quad (1)$$

where P is the city population and D is the distance from the city center to the observer. Now, the night sky brightness, I , is proportional to the city luminosity, L , or

$$I \propto L. \quad (2)$$

Further, Walker³ showed that there is a luminosity-population relationship,

$$L \propto P, \quad (3)$$

and used his improved measurements to show that the brightness follows the relation

$$I \propto D^{-2.5}, \quad (4)$$

where I is the night sky brightness due to city lights. Combining these relationships gives

$$I \propto P D^{-2.5}. \quad (5)$$

Since electromagnetic radiation follows a D^{-2} law and extinction through the atmosphere modifies this somewhat, we can more generally state the above equation as

$$I = C P D^\alpha, \quad (6)$$

where C and α are empirical constants that vary slightly as function of population. Treanor⁴ extended this to consider scattering by molecules and aerosols through a homogeneous atmosphere, resulting in

$$I = a P (b D^{-2} + c D^{-1}) e^{-\kappa D}, \quad (7)$$

where a is a constant relating population to urban development for individual cities, b and c are constants, and κ is the aerosol extinction coefficient. Berry⁵ modified Treanor's relation to model the zenith brightness of small cities as proportional to the square root of their population:

$$I = a' \sqrt{P} (b D'^{-2} + c D'^{-1}) e^{-\kappa D'} \quad (8)$$

where $D' = \sqrt{(D^2 + h^2)}$ and h is the height of the scattering layer.

Garstang^{6,7}, whose model will be explained in detail below, greatly extended these models by considering more detailed geometrical and physical considerations. Garstang's improvements included 1) generalizing the city as a uniform disk rather than a point source, thereby allowing for an estimation of the illumination closer to the city; 2) using an exponentially decreasing atmosphere, with differing scale heights for molecules and aerosols; 3) adding an atmospheric "clarity" parameter to represent the number of aerosols; 4) allowing for an angular distribution of light emission (light shields); 5) accounting for ground reflection using a Lambertian distribution; 6) including models for the night sky background; and 7) correcting for curved earth geometry. Garstang's model can be expressed by,

$$I = a P U D^{-2} (DS) (EF), \quad (9)$$

where DS accounts for aerosol scattering between the city and an element of atmosphere, and EF is an integrated extinction factor. Here, a has the same meaning as in equation (7), and U is a constant.

Finally, Aubé, et al.⁸, have taken a somewhat different approach and have developed a research grade 3D heterogeneous light pollution model in which a city may take any shape, can have a variable distribution of light sources along with local variations in topography and ground reflectance. They have coupled their model with in situ light pollution measurements in order to determine the aerosol optical depth at night using an iterative method employing these measurements. The model includes spatial heterogeneity – in lighting geometry, in lighting spectral dependence, in ground spectral reflectance, in topography; it also computes 1st and 2nd order molecular and aerosol scattering, as well as aerosol absorption. In Aubé's model, the spectral light intensity, as received by an arbitrarily placed spectrometer, is

$$I \approx I_1 + I_{r1} + I_2 + I_{r2}, \quad (10)$$

where I_1 is the single scattered intensity, I_{r1} is the first scattered intensity after reflection on the ground, I_2 is the second order scattering intensity, and I_{r2} is the second order scattering intensity after reflection on the ground.

All of these models have various strengths and shortcomings. Walker's model is empirical. Treanor's and Berry's model consider cities as point sources, the former assuming a homogeneous atmosphere. Berry's fit of the luminosity-population relation to the square root of the population has been shown⁶ to be an effect of inclusion of satellite cities. Garstang's model, while the most comprehensive of the semi-analytical models, does not include the effects of cloud cover; and computer run-time is significant. Finally, while Aubé's research-grade model does include all significant lighting and atmospheric effects, it is computer intensive and omits

calculations where atmospheric absorbers are present. Since in our case the desired end product will be used for determination of light levels for Army sensing, the model must account for all Army relevant effects (lighting and atmospheric) and also be quick running. With some judicious changes, Garstang's model, discussed in some detail in the following, is the primary candidate.

Garstang's Model

Garstang's model estimates city brightness as a function of distance, look-angle, and city population. It assumes a mostly clear atmosphere, but can be modified to include other atmospheric conditions (foggy, hazy, etc.) and it has been shown⁷ to reproduce the observed brightness values for a wide array of cities and geometries. Garstang models the city as a uniform circle, rather than a point source which gives better results for observers near the city and can also be used for determination of the sky brightness from within the city as well.

Geometry

The city is modeled as a circular area of uniform brightness on a flat earth. Figure 2 presents the geometry used in Garstang's model and in the equations below.

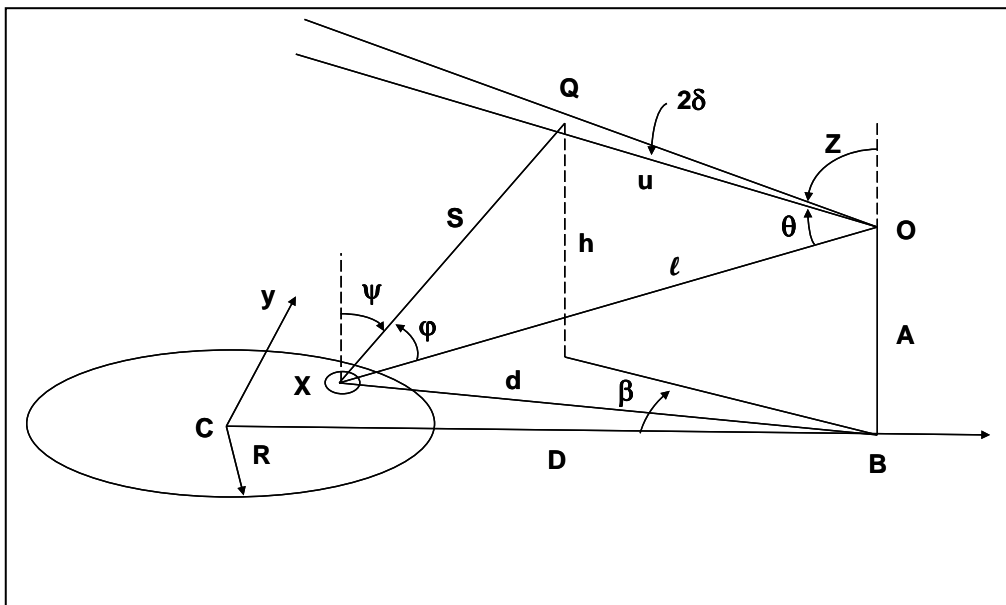


Figure 2. Scattering of light by a city. Light from a small area at X is scattered at Q and received by the observer O. Q is at a height h above the plane of the city, β is the azimuth of Q, and Z is the zenith distance of observation. The observer receives light from a cone of semiangle δ around QO (from Garstang 1986).

Light Output and Distribution

The model assumes that the artificial lighting produces an output, L lumens per head of population, so that the total city light output is L P lumens, a fraction of which, F, is radiated directly into the upper hemisphere: (1-F) being radiated downward toward the ground. Of this

(1-F), a fraction G is reflected upward with a Lambertian distribution. The value of G is dependent upon the cultural level of the city (mostly dirt roads vs. heavily paved with concrete and asphalt) and of the weather, most notably, snow cover.

Atmospheric Density Structure

The model also considers the effects of an inhomogeneous atmosphere, in-so-far as the densities of both molecules and aerosols decrease exponentially with height, with different scale heights for each. The assumption of an exponential atmosphere allows the integration along the observer's line-of-sight to be carried out, thereby providing brightness as a function of the observer's zenith distance. The molecular density profile is given by

$$N_m(h) = N_m(0) e^{-cH}, \quad (11)$$

where $N_m(0)$ is the density of atmospheric molecules at sea level, c is the reciprocal scale height of the molecular atmosphere, and H is the altitude of the ground above sea level.

The aerosol density profile is given by a similar expression

$$N_a(h) = N_a(0) e^{-aH}, \quad (12)$$

where N_a and $N_a(0)$ have the same relative meanings as in the molecular case. The reciprocal scale height for aerosols, a , is given by

$$a = 0.657 - 0.059 K. \quad (13)$$

Here K is an independent parameter measuring the ground level ratio of aerosol scattering to molecular scattering:

$$K \propto N_a(0) \sigma_a / [N_m(0) \sigma_R e^{-cH}] \quad (14)$$

where σ is the aerosol (a) or Rayleigh (R) scattering cross section. Thus, K is an indicator of atmospheric clarity: when $K = 1$, then $N_a(0) \sigma_a \propto N_m(0) \sigma_R$, or the aerosol content is roughly equivalent to the molecular content of the atmosphere. The proportionality constant is determined by using a fairly clear sea level atmosphere.

Scattering by Molecules and Aerosols

Garstang, following Treanor and Barry, considers the light received at the observer's zenith to be comprised of 1) the attenuated direct beam from the city lights, 2) light singly scattered from the direct beam along a narrow cone in the direction of the observer's zenith, 3) attenuation by the atmosphere and 4) scattering by the element of atmosphere at Q towards the observer's zenith at O . Under these conditions, the representation of scattering by aerosols and molecules, the DS term in equation (9), is represented as

$$DS = 1 + N_a(0) \sigma_a \{ 1 - \exp(-a S \cos \varphi) / (a \cos \varphi) \} + R_c, \quad (15)$$

where S is the distance from the along path XQ , ψ is the zenith distance, and R_c is a correction factor to allow for double scattering by molecules. Implicit in the derivation of DS is a forward scattering approximation, i.e. since a great deal of scattering by aerosols at visual wavelengths is predominantly along the light beam's axis, a small-angle approximation was utilized in the derivation of equation (15).

Brightness Distribution

Referring to figure 2, and choosing CB as the x axis; then light from an element of area $dx dy$, at X , travels to Q . Single scattering is considered along the XQ path. The combined contribution of the direct beam and the single scattering is again scattered at Q , some of which is received by the observer at O . After some geometrical manipulations, it may be shown⁶ that the brightness observed at O in the direction of Q , in units of lumens $\text{cm}^{-2} \text{sr}^{-1}$, is given by

$$B = \pi N_m \sigma_R \exp(-ch) \iint (dxdy / \pi R^2) \int_0^x I_{xy} s^{-2} (EF)_{xQ} (EF)_{QO} (DS) \times \{ \exp(-ch) 3(1 + \cos^2 \Theta) / (16\pi) + \exp(-ah) 11.11 K f(\Theta) \} du, \quad (16)$$

where $\Theta = \theta + \phi$, and $f(\Theta)$ is an analytic representation of a typical aerosol phase function, and all other symbols either appear in figure 2 or have been previously defined. The expression $I_{xy} s^{-2} (dxdy / \pi R^2)$ represents the flux per unit area falling on the scattering volume $\pi \delta^2 u^2 du$ at Q from the area $dx dy$ of the city.

Modifications for Army Relevance

In terms of Army applications, Garstang's model has some deficiencies, notably in city lighting and in computer run-time. The first is a database problem, which may be solved by Cinzano's atlas or other means, and the second can be addressed by parameterization of particular functions contained in Garstang's code. One method for addressing the problem of computer run-time is by parameterization of the various functions in Garstang's code; this will be discussed in some detail below.

Garstang's formulation for the brightness, equation (16), requires considerable computation. A simplified solution, used in TAWS, produces a parameterization of this model by using equation (6) in conjunction with computations from Garstang's model. Thus, we can parameterize both C and α by using results compiled by Garstang⁷ reprinted here as table 1. P is the city population and C depends on light emission per head of the population and the reflectivity of the ground⁶.

Table 1. Values of log C and α as a function of population for large distances.

log P	log C	α
3.0	-1.29	-1.90
3.5	-0.96	-2.19
4.0	-0.56	-2.48
4.5	-0.22	-2.69
5.0	0.22	-2.94
5.5	0.78	-3.23
6.0	1.91	-3.77
6.5	3.61	-4.50

For a given population, log C and α are interpolated from the data points shown here and used in equation (6). However, these results would have to be modified for both closer distances and for various values of Garstang's K parameter (K = 0.5 was used by Garstang in compilation of table 1).

Using equation (6) in conjunction with table 1, we can parameterize directional dependencies, as seen from point O, for the city light intensity by making two assumptions to the simple zenith brightness: a Gaussian drop-off in intensity in the azimuthal direction and a secant function in the zenith direction⁴.

The brightness-distance relationship does not include a model for the angular effects on observed brightness; rather it is used to compute brightness in the zenith direction only. Berry⁵ presents measurements showing that city sky brightness, for zenith angles less than 80°, decreased proportionally as the secant of the zenith angle. Thus a function for variations in zenith angle may be used

$$f(z) = a \sec(z), \tag{17}$$

where a is a constant of proportionality determined from empirical data and z is the zenith angle.

Using the data from Berry, Walker and Garstang, a best-fit secant function with its corresponding constant of proportionality, a, can be determined as a function of population, and is presented in table 2.

Table 2. Estimates of the constant of proportionality, a, and the corresponding population category.

Population Category	Constant of proportionality, a
P ≤ 2000	0.4
2000 < P ≤ 6500	0.7
6500 < P ≤ 20000	0.9
20000 < P ≤ 65000	1.2
65000 < P ≤ 200000	1.6
200000 < P ≤ 650000	2.0
650000 < P ≤ 2000000	2.4
2000000 < P.	2.9

For the range 80–100 degrees, the value of the secant law valid at 80 degrees is used. Beyond 110 degrees a modified secant law is used to ensure that the value of the zenith angle correction approaches zero as the zenith angle approaches 180 degrees. Between 100 and 110 degrees, an interpolation is performed. In all cases where the secant angle is greater than 90°, care must be taken to insure that $f(z)$ does not go negative.

For the azimuthal variations in the urban brightness levels, TAWS uses a Gaussian weighting function to model that drop off:

$$f(\beta) = e^{\left(\frac{-\beta^2}{2F\sigma^2}\right)}, \quad (18)$$

where β is the azimuthal angle as presented in figure 1, σ^2 is the variance, and F is a multiplicative factor that produces a new effective variance. Based on qualitative observations, TAWS modifies the variance of the Gaussian weighting function as a function of zenith angle. F is set equal to $\sec(90-z)$, resulting in a larger effective variance for small zenith angles and decreasing as the zenith angle approaches 90 degrees. This ensures that the azimuthal dependence of the sky brightness decreases as one looks toward the zenith. A graph of the azimuth angle correction function for three sample zenith angles is shown in figure 3.

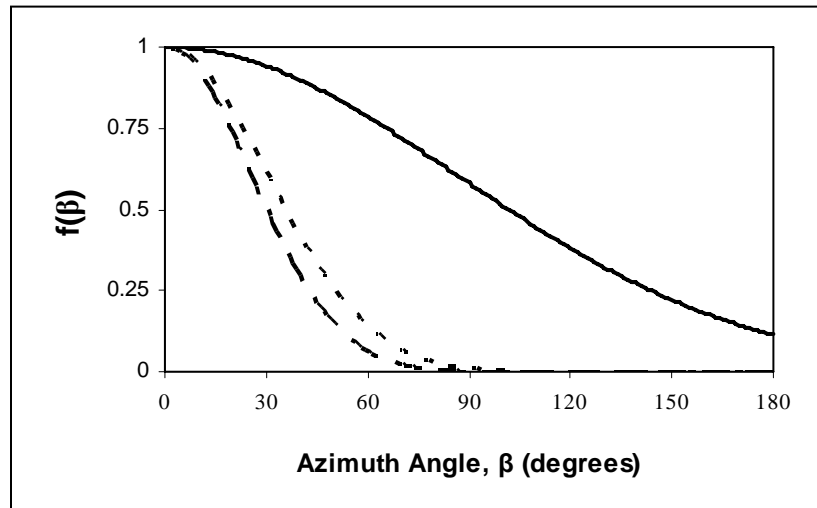


Figure 3. Azimuth angle correction function, $f(\beta)$, for three zenith angles: $Z = 5^\circ$ (solid), 45° (short dash), and 90° (long dash).

After identifying these factors for the change in brightness due to zenith and azimuth angle effects, the broadband brightness from the city is calculated by:

$$B = \text{CPD}^\alpha f(z) f(\beta). \quad (19)$$

Summary

As the reader may have noticed, the simplistic model discussed above implicitly restricts the model to distances far from the city ($D \gg h$). Other assumptions, particularly the azimuthal

falloff of city brightness, should be examined and strengthened by running the full Garstang model and comparing those results with these. Other items that must be examined are the applicability of obtaining either population data or city source illumination data in some fashion that does not require the user to consult an atlas. Coupling population estimates in a tabular form or using an underlying map with Cinzano's brightness atlas are possible avenues of approach that will be examined. Additional areas of work will be adding additional aerosol types, using appropriate phase functions rather than a generic phase function, and cloud cover.

The model is currently in the preliminary stages. If the reader believes that such a model could be of use in their program, they are urged to contact the author – at this early stage, the model can be constructed to accommodate various uses.

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